SPECTRAL SIMULATION FOR CULTURAL HERITAGE

A Scientific Methodology and Some Examples

BY

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Patrick Callet has great experience of teaching in many schools and colleges. He is head of several projects concerning photonics and rendering with many academic and industrial partners as well as digital heritage projects (CALLISTO-SARI, LIMA, GEOPAST and CoLuDoRAM, Royaumont, Idalion) involving important institutions.

He collaborates with renowned institutions in France such as prestigious museums (Louvre, National Museum of Asian Arts, Saint-Denis Basilica, Bibliothèque Nationale de France, Fondation Royaumont) and several schools of engineering. His research fields mainly concern physically based rendering and colour.

He is French delegate member at AIC and Associate Editor of the international journal *Color Research and Application* (Wiley), he is also vice-president of the *Centre Français de la Couleur* and a member of the CIE (TC1 72A) Technical Committee. Author of the famous book written in French, *Couleur-lumière, couleur-matière – interaction lumière-matière et synthèse d'image*, 1998, Diderot. Co-author of several books and many international articles and scientific movies.

See also: www.dailymotion.com/fr/relevance/search/lutopix/1

For a picture of Patrick Callet, see page 28.

The Colour Group (GB) has established a series of
Occasional Publications of which this is the third.
The first was *Emulous of Light: Turner's Colour Revisited*
by Dr John Gage (2010) and the second was *Chevreul's Colour Theory and Its Consequences for Artists* by Dr George Roque. Both are available from
http://www.colour.org.uk/archive.html

Abstract
We present a set of models and results obtained in a general research framework on the physically-based rendering of cultural heritage objects or monuments. We show how it is possible to visualise some plausible restitutions of how an object might have appeared many centuries ago. We present several examples from studies on bronzes, the metallic aspect, polychromy and gilt, natural lighting and spectral simulation. The first part deals with metallic reflection from silver-lead alloys used in Mesopotamia 22 centuries ago. The second part discusses some questions and models related to the rendering of polychromy, gilding and natural lighting effects during the medieval period. Thanks to the simulation of global illumination in spectral rendering operating with the photon mapping algorithm, a new interpretation of polychromy effects is proposed, including the lightguide effect produced by gilding. Measurements were made on formulated polychrome samples and natural lighting at sunset to validate the new hypothesis.

1. Introduction

For many years our scientific activity has concerned two main fields where colour and rendering are linked. Physics is at the heart of our method of research. The modelling of the interaction between light and materials is concentrated around a crucial notion that links fundamental properties of materials to their visual aspect in specific lighting conditions. All the realised projects were undertaken with an interdisciplinary team of chemists, physicists, engineers, computer scientists, and video-makers.

The first project described in this paper concerns biophotonics and its application to bio-inspired pigments and paints (LIMA project). The second is dedicated to cultural heritage comprising objects and monuments and investigates the rendering of large 3D scenes (several millions of triangles in each mesh). The last project concerns the Tablet of Idalion (Cyprus), a small bronze, which has been replicated via a 3D model, in the same physical composition as the original tablet. All these projects have involved partnerships with prominent French museums such as National Museum of Asian Arts - Guimet, Saint-Denis Basilica, Fondation Royaumont, Louvre, Bibliothèque Nationale de France and a small museum in Dali (Cyprus). These projects were all rooted in the study of fundamental optical properties of materials, and they enabled us to acquire invaluable experience in the rendering of museum artefacts and polychrome monuments.

A crucial principle when rendering the effects of the interaction of light with materials is to separate spectral properties from geometrical properties. In other words, the distinction is made between the intrinsic and extrinsic properties of materials. Intrinsic properties arise from the complex dielectric function of an homogeneous material. Extrinsic properties are linked to the state of the surface or conditioning of the material (powders, rods, tubes, in solution or inclusion, bulk or in thin film, etc.). In many instances the materials encountered for colouring artworks are organised in layers (paintings, gilding, stained glass windows, etc) of different thicknesses. Modern pigments are multilayered systems and differ at different scales from layered paints or glazes.

We present two studies that have been at the foundation of the field known today as Virtual Metallurgy. Applying the previous models dedicated to metals and some phenomenological approaches for powders and paints, we then tackle the question of the rendering of polychromy effects with gilding. Then we show how medieval painters, sculptors and architects worked together to produce some special effects with natural light in very close relationship with the Christian symbolism of light.

2. Optical considerations

We start with definitions of intrinsic and extrinsic properties of homogeneous materials. The essential principle for modelling the overall optical properties of any material lies in the components of the complex dielectric tensor. The most useful form is the complex index of refraction, defined as:

$$[n'(\omega)]^2 = \varepsilon(\omega)\mu(\omega) \qquad (1)$$

where ω is the wavelength of the incident wave, $\varepsilon(\omega)$ is the dielectric permittivity, and $\mu(\omega)$ is the magnetic permeability. It is usually admitted that the last term is equal to unity over the entire visible spectrum, but next generation materials should pay greater attention to that term. In computer graphics the complex index of refraction is commonly written in terms of wavelength dependency:

$$n(\lambda) = c/v_{\Phi} \qquad (2)$$

where c denotes the velocity of light in vacuum, while v_{Φ} is the phase velocity of the electromagnetic incident wave inside the medium of propagation. It is important to recall the definition in terms of relative velocities to explain metallic reflection. In spectral simulation we are concerned by all the effects due to the interaction of light with materials. *Virtuelium*, our free software, is briefly described below. It still cannot account for uniaxial or biaxial materials. These more complex aspects will be considered in the foundation of Virtual Mineralogy with applications to gemmology. For the most common materials encountered in cultural heritage projects the mean complex index of refraction is relevant. Optical anisotropy will not be considered here. The Max Born [4] notation for the index of refraction is used:

$$n'(\lambda) = n(\lambda)[1 + i\varkappa(\lambda)] \qquad (3)$$

where $n(\lambda)$ is the real part of the index of refraction and $n(\lambda)\varkappa(\lambda)$ is its imaginary part. Some indices of refraction for transparent materials, for which $\varkappa(\lambda) = 0$, are given in Table 1.

Table 1: Some indices of refraction for transparent materials

Name or Chemical formula	**Real optical index exact or mean value for the sodium D light**
Vacuum	1.00000
Air (dry)	1.00027
Water	1.33240
Na_3AlF_6 (cryolite)	1.33800
MgF_2 (sellaïte)	1.38200
SiO_2 (amorphous silica)	1.46010
KCl (sylvite)	1.49000
Polymers	1.4-1.6
Crown glass	1.52200
Light Flint glass	1.54300
$NaCl$ (halite)	1.54400
Mica	1.60000
Flint glass	1.60700
MgO (periclase)	1.73500
Dense flint glass	1.74600
Al_2O_3 (corindon)	1.76500
Scale fish	1.85000
$PbCO_3$	2.10000
$BiOCl$	2.15000
ZrO_2 (baddleyite)	2.16000
C (diamond)	2.41800
TiO_2 (anatase)	2.50000
$CaTiO_3$ (perovskite)	2.74000
TiO_2 (rutile)	2.75500

The index of refraction varies only very slowly over the visible spectrum. For understanding of its influence on chromatic effects, we present in Fig. 1 and Fig. 2 a visual comparison of the dispersion of light through three transparent materials: two glasses and diamond. Coloured materials are also characterised by an absorption index. For very small values of those indices all coloured glasses or translucent materials (solutions, polymers, etc.) can be produced. The effect of electronic transitions inside the valence band of the materials is to increase the imaginary part of the complex index of refraction. These transitions modify the real part of the index because the two spectral functions (optical index and absorption with semi-conductive and conductive materials including metals, alloys, metallic sulphides PbS, FeS_2, etc.) will provide the highest absorption values.

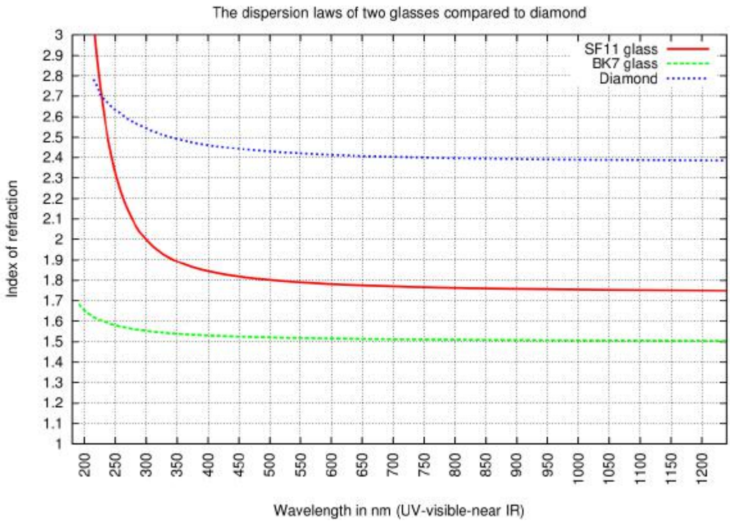

Figure 1: Comparison of three dispersion curves of transparent materials on the UV-visible-near IR interval. Two glasses, BK7 and SF11, are compared to diamond. It is obvious that all the indices have higher values in the UV region and consequently a higher power of refraction for that part of the spectrum.

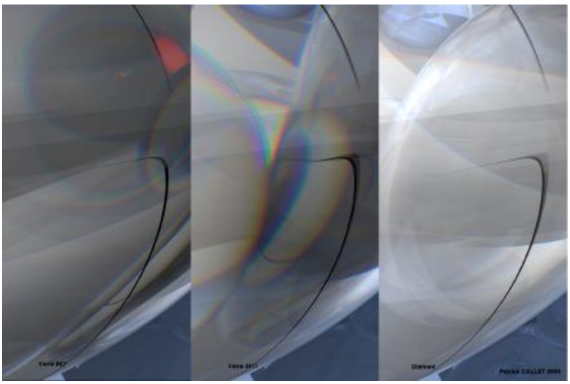

Figure 2: Using the dispersion curves shown in Fig. 1 the computation in spectral ray-tracing illustrates the internal caustics produced in exactly the same conditions of viewing, lighting, state of surface, etc. The computation involves 81 spectral bands of 5 nm width over the interval [380;780] nm. CIE D65 illuminants, CIE 1964 supplementary reference observer (10°).

As the ideal reflectivity of a homogeneous material depends on its index of refraction, we can represent mathematically the variation of the mean reflectance factor for any complex index of refraction for an uniform lighting (orthotropic distribution) over the local tangent plane of a surface. The computation involves a spatial and spectral integration of the ideal reflectance factor, here for an unpolarised incident light. The result of this double integration leads to a 3D surface of optical properties. In the cultural heritage field, it is necessary to account for materials that are very irregular in both their optical properties and their state.

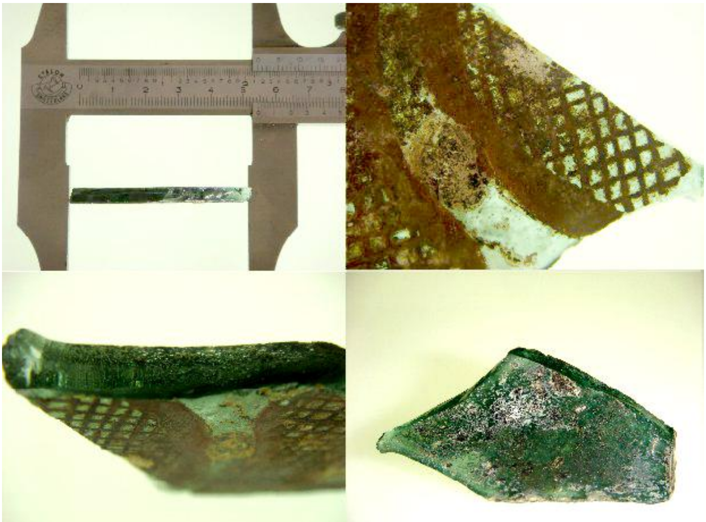

Figure 3: Several images of a piece of stained glass window found in Royaumont Abbey (cistercian church at 40 km to the north of Paris destroyed during the French Revolution of 1789). The state of surface of the glass is not only due to alterations during many centuries but also due to the 'grisaille' (the painted drawings). The complexity is increased by the presence of small bubbles inside the sheet of glass on the external side, probably handcrafted manufacture. Highly transparent glasses were not still elaborated in the XIIIth century. We also measured the absorption of the glass in several regions of the sample by spectrophotometry.

An example is given in Fig. 3 where the many levels of complexity are visible. One of the main difficulties lies in the determination of the complex index of refraction of the medieval glass. To do this, we use spectroscopic ellipsometry which requires a special preparation of the samples to fulfil the ideal Fresnel conditions. Spectroscopic ellipsometry consists in the inversion of the Fresnel formulas. As

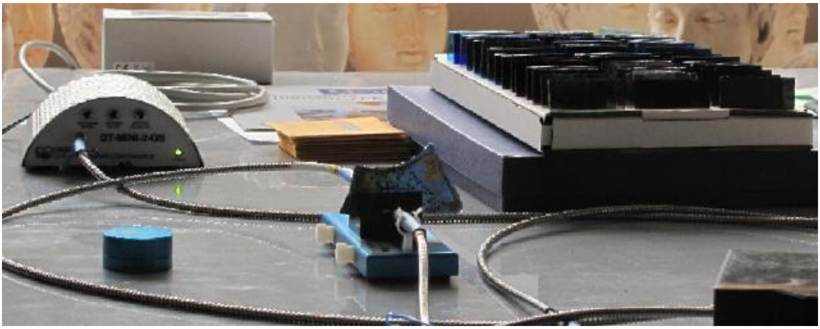

Figure 4: Spectrophotometric measurement of the transmitted light by a medieval glass sample. A set of medieval-like glasses are used for visual comparison with the authentic sample and to measure their optical constants by spectroscopic ellipsometry. That work is in progress for retrieving the most probable natural lighting in the abbey-church of Royaumont. Here is shown a sample in the measuring head (centre) on the bench with light source, fibre optics and standard samples (right).

these formulae explicitly depend on the complex index of refraction, the measurement of the state of polarisation of the reflected light and the amplitude ratio of the parallel and perpendicular components to the plane of incidence for the reflected light leads to the complex index of refraction [8].

Generally, the medieval glasses used in our study cannot fulfil the Fresnel conditions and they do not have a homogeneous internal structure. For attempting to acquire the optical data we can compare the transmission spectra of a set of medieval-like glasses for which we can measure n and $\varkappa$ by spectroscopic ellipsometry (Fig. 4 and Fig. 5). The results by simulation obtained from these measurements are shown in Fig. 6; the commercial names are reported on the simulated samples and the variations of the complex indices of refraction are given in the right part of the figure for two glass samples. Notice the difference in magnitude between the two absorption indices reported here. Notice also the last line of the left image where the thickness of the samples is virtually increased. Until now it was not possible to account for the influence of extrinsic parameters on visual appearance. These are geometrical parameters, such as roughness at the wavelength scale and waviness at the scale of mm. The corresponding surface states can be rendered with many algorithms. The last very common effect is based on a visual illusion known as ‘bump mapping’, which perturbs the local normal by using an image-map, computed for each pixel affected. An example of four levels of roughness is given in Fig. 11.

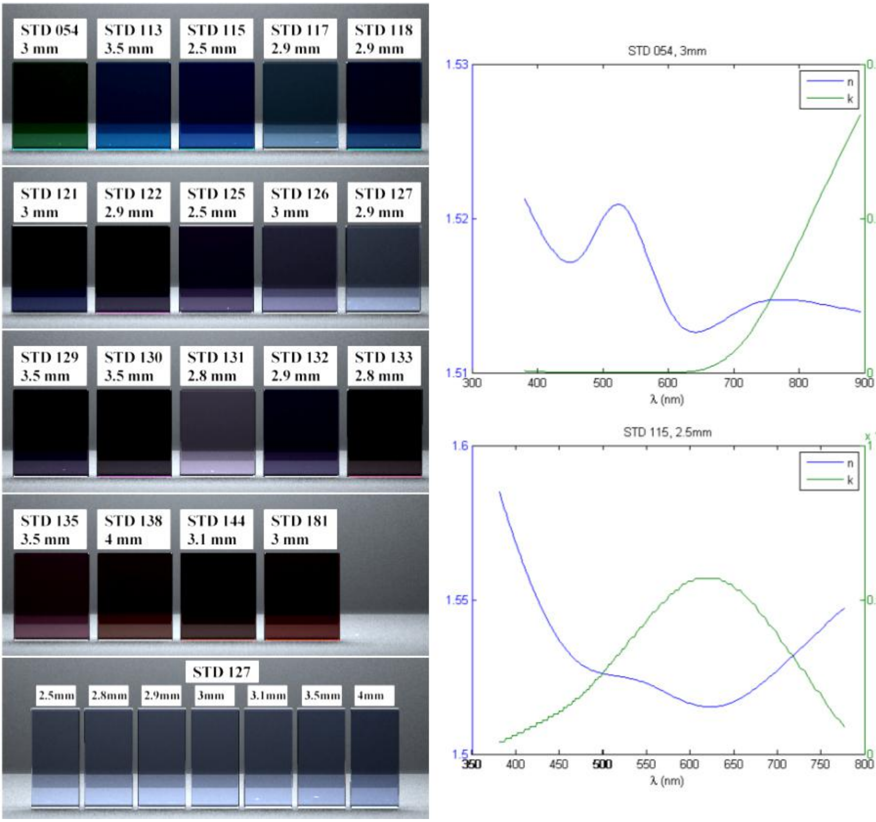

Figure 5: Spectral simulation with Virtuelium of the optical properties of some samples of medieval-like glasses from the glassmaker Saint-Just. The samples are virtually disposed inside a virtual light booth corresponding to the real light booth (GTI-ColorMatcher) available in our lab. The real emission spectra of these light sources and the diffuse reflectance spectrum of the paint were also recorded with a MAYA 2000 spectrophotometer (Ocean Optics). The complex indices of refraction were acquired by spectroscopic ellipsometry and are drawn in the right part of the figure. The simulation then corresponds to the same conditions except for the virtual observer! The supplementary reference colorimetric observer 1964 (10°) is used.

Figure 6: Spectrophotometric measurements of the transmitted light by a medieval stained glass window at LRMH (Laboratoire de Recherche des Monuments Historiques). We have also used several light sources.

3. Metals and alloys

The high conductivity of metals and their opacity, even for thin films, is explained by the high values of their absorption indices. The general principle is very well described by plasma physics. Working with only optical constants of the elements [22], the real and imaginary data *n* and *k* are classified in terms of energy expressed in eV units. The visible spectrum corresponds to the range [1.7; 3.3] eV of incident electromagnetic waves. The data are then sorted in the reverse order of wavelengths from the red to blue regions. A computer simulation in spectral rendering with Virtuelium is shown Fig. 7 where only the available elements are represented. Notice that the gases and all the non-metallic elements are not computed except for carbon in its crystalline diamond form. A few pictures obtained with the first versions of Virtuelium are visible on the Internet, included by Mark Winter at the university of Sheffield[1].

Visualisation of the elements in the classical periodic table is shown in Fig. 7. More fundamental modelling *ab initio* is currently in progress. The challenge is to predict the complex components of the dielectric function given only the electronic structure of the elements. Andras Vernes [24, 25] obtained remarkable agreement between optical constants of platinum with the recorded data of Palik [22]. The

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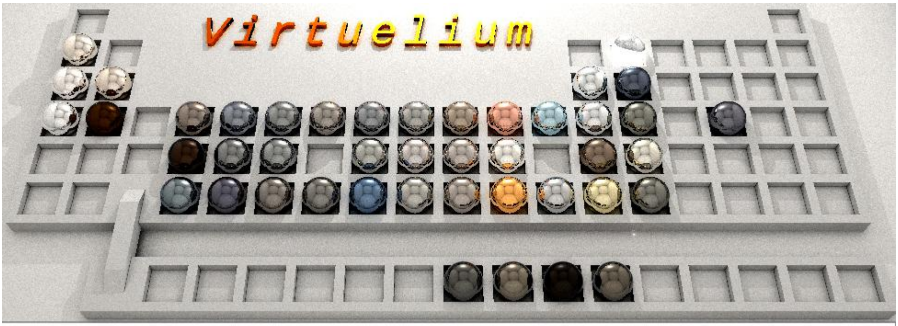
1 www.webelements.com/germanium/physics.html

absorption index clearly depends on the density of free electrons brought by each metallic agent of the alloy. The optical properties are then modified by physical parameters involving braking forces when the electrons attempt to cross the local boundaries of their own crystallisation region (granular alloys). The electronic displacement depends on the electric field applied to the alloy. At optical frequencies, we observe that the plasma frequencies of each species are of the same order. A special property linked to conductivity lies in the possibility for very highly conductive metals to have $n(\lambda) < 1.0$ while $\varkappa(\lambda)$ reaches values of 10 up to 60 and even more.



Li												**C**	
Na	**Mg**										**Al**	**Si**	
K	**Ca**	**Ti**	**V**	**Cr**	**Mn**	**Fe**	**Co**	**Ni**	**Cu**	**Zn**	**Ga**	**Ge**	**Se**
		Zr	**Nb**	**Mo**		**Ru**	**Rh**	**Pd**	**Ag**		**In**	**Sn**	
		Hf	**Ta**	**W**	**Re**	**Os**	**Ir**	**Pt**	**Au**	**Hg**	**Tl**	**Pb**	
								Gd	**Tb**	**Dy**	**Ho**		

Figure 7: Top: the periodic table, in its conventional presentation, containing several metallic and semi-metallic elements plus carbon in its crystalline variety of diamond, are simulated. The rendering in global illumination by spectral photon-mapping only, uses the complex indices of refraction. For those elements having anisotropic structure we use an appropriate mean index depending on the crystalline structure. CIE-D65 illuminants, null roughness, supplementary colorimetric observer CIE 1964 (10°). Bottom, the more virtual and less visual presentation of the periodic table.

A Chinese horse statuette and the tablet of Idalion, two bronzes of great symbolic importance, have been studied recently. The horse statuette (see Fig. 8) provided for the first time the opportunity to render all possible true bronzes (strictly Cu and Sn alloys) according to their metallurgical composition. An interpolation was made of the complex dielectric functions, not of colour or reflectance or even of the index of refraction. The tablet of Idalion, a Cypriot bronze including an exceptionally weak amount of tin (2.4 %) is shown in its original optical state in Fig. 10 obtained by spectral simulation after an accurate 3D digitisation of the original tablet on display in the Bibliothèque Nationale de France in Paris.

Figure 8: Metal and lighting. Ambient lighting and directional lighting are compared between a real object (upper line) and a simulation of the chinese bronze statuette. It clearly appears that a uniformly lighted metallic object cannot be perceived as metallic if there are no directional light sources in the scene. (Photos: R. Sève ; Computed images: P. Callet). CIE D65 illuminants and colorimetric reference observer 1964 (10°).

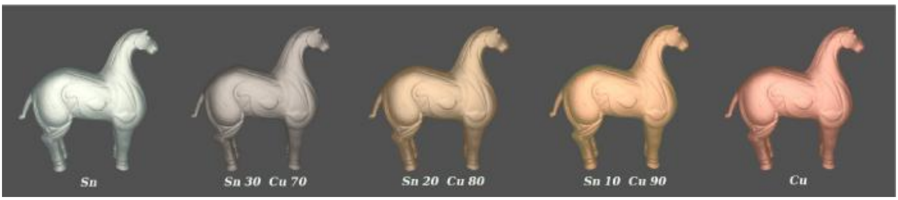


Figure 9: Virtual metallurgy of bronzes. The rendering of the statuette, thanks to Virtuelium, with no influence of any environment, is computed with the measured optical constants acquired by spectroscopic ellipsometry using the physical samples of bronzes prepared at LGPM (Laboratoire de Génie des Procédés et Matériaux) by Anna Zymla. Moderate roughness (Beckmann model) CIE D65 illuminants and colorimetric reference observer 1964 (10°).

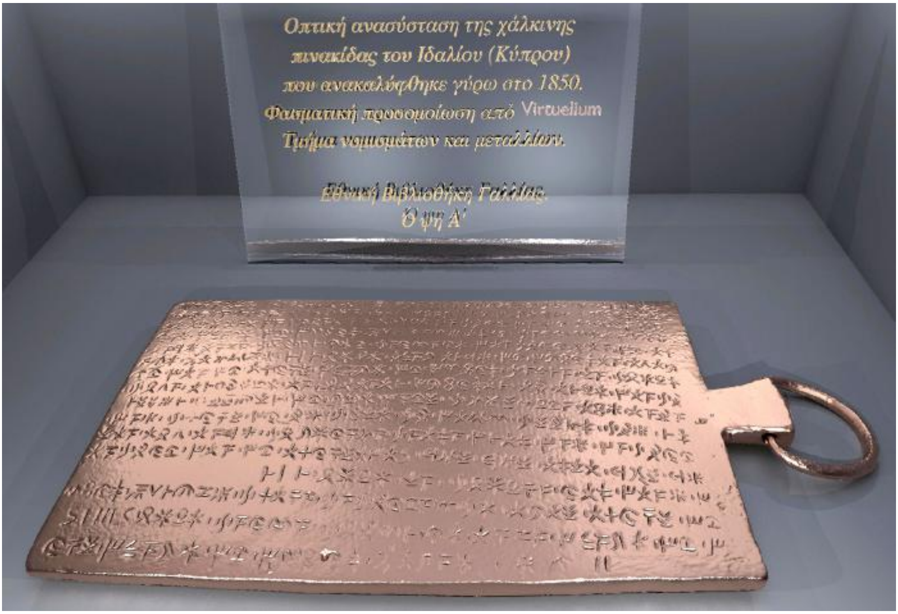


Figure 10: Virtual metallurgy of bronzes. The rendering of the tablet of Idalion, thanks to Virtuelium, placed in a virtual light box. A moderate roughness is applied to retrieve the original optical appearance. This replication is not only virtual but also physical in 'real material' and exhibited at the archæological museum in Dali (Cyprus). CIE D65 illuminants and colorimetric reference observer 1964 (10°).

The physical replica made in France (Fig. 12) has been exhibited at the archæological museum of Dali in Cyprus since 11th November 2010. One can see the technological prints very similar to curves of level indicating the modern lost-wax process employed for bronze casting at the Foundry in Sèvres (CTIF). We decided to leave these visible prints for testifying the replica, and to avoid confusion with the green-brownish original appearance of the now-corroded tablet. The movie *Facsimilé* describes all the steps of the project of virtual and physical replication of this important part of the archæological heritage of Cyprus. Our replica is very well matched to the present shape of the original tablet, but has the plausible appearance of the original as viewed 25 centuries ago. Fig. 11 shows how it would look with four different levels of surface roughness. The vertical presentation in Fig. 12 recalls that this object was originally hung inside the temple of Athena on the top of the hill of Idalion.

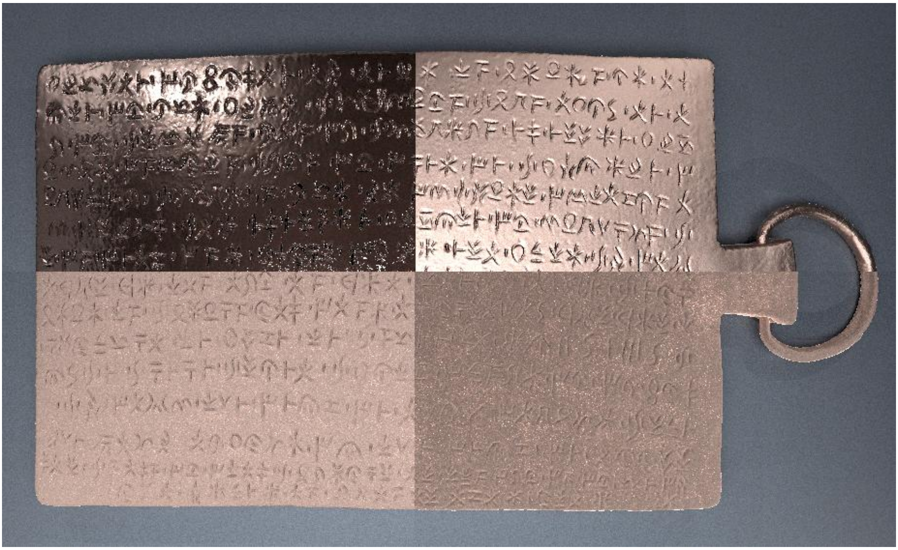


Figure 11: Virtual metallurgy of bronzes. The rendering with Virtuelium of the influence of the state of surface of the tablet of Idalion, placed in a virtual light box. Four levels of roughness are rendered for 0.0, 0.3, 06 and 0.9. CIE D65 illuminants and colorimetric reference observer 1964 (10°). Notice how the lightness is varying due to the surface scattering when increasing the roughness.

The complex indices of refraction were acquired by spectroscopic ellipsometry (see next section for more details) from five physical samples of Cu, Sn and three alloys (10, 20 and 30% of tin concentration in copper) specially prepared and studied or verified by SEM. The obtained results, in computed images, are shown in Fig. 9. Colour and metal, here for bronzes in a large meaning, are described versus the metallic composition in Tables 2 and 3.

Table 2: Examples of bronze composition

	% Cu	% Sn	% Pb	% Zn	% Fe	% Sb, As
Greek bronze	88	6-9				
Roman bronze	90	3	3			
Chinese bronze	80	4	10	2	4	
Colonne Vendôme	90	6	3	0.2		0.1 - 0.86
Bronzes for bells	74 - 77	23 - 25	0.3 - 2.5	0.3 - 0.6	0.3 - 0.6	0.2 - 0.4
Bronzes for mirrors	62 -71	23 - 32	5 - 7			

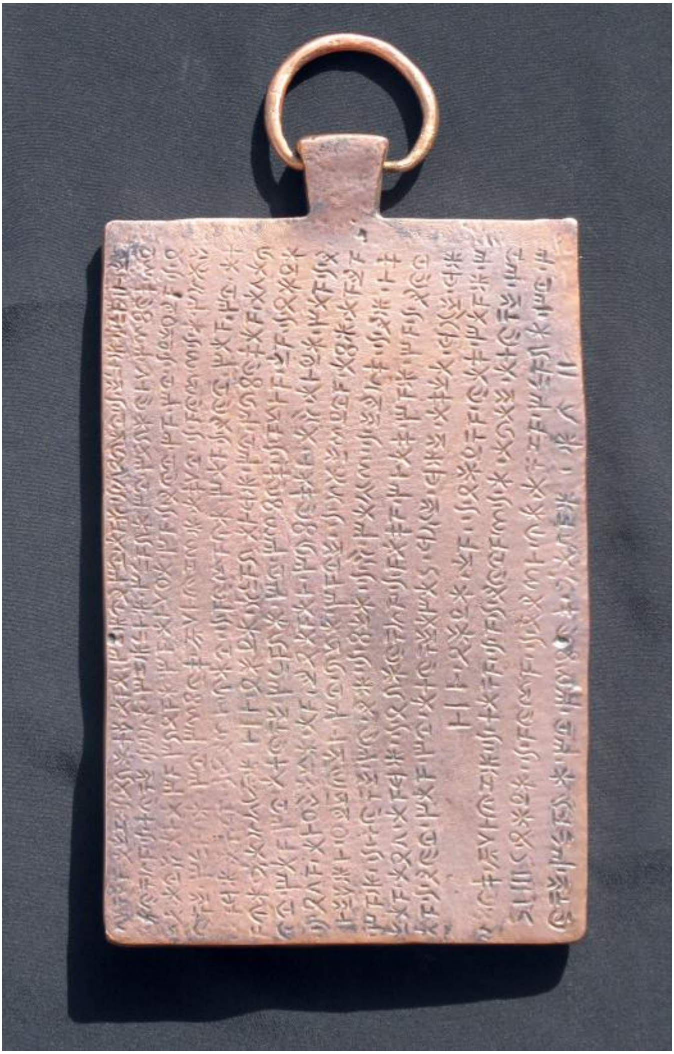


Figure 12: Real metallurgy of bronze. The bronze replica obtained in 2010 as it could have appearede to the people of Cyprus in Idalion 25 centuries ago. The optical aspect is slightly modified by the natural patination of the alloy inside its display cabinet at the museum. The image is taken in Dali before the official handover ceremony under the natural sun light of Cyprus.

Table 3: Usual colour denomination vs tin concentration in bronzes

% Tin	Colour	% Tin	Colour	% Tin	Colour
0	Red	0-5	Red	5-15	Gold
$\geq$ 15	Light gold	30	Grey with a pink shade	$\geq$ 32	Grey blue
100	White silvery				

3.1 Importance of the index of refraction

As previously mentioned, the complex index of refraction notion appears to be more important than spectra for modelling appearance. Our arguments for this approach are founded on the following observations:

- opacity and very high reflectivity of metals are characterised by the absorption index *k*, while the real part of the refractive index can be less than unity (evanescent wave) e.g. noble metals;
- semi-transparency in thin slabs and a high reflectivity of semi-metals are characterised by a weaker absorption index *k*;
- transparency and translucency are characterised by a completely real refractive index associated with a very weak spectral absorption index *k*.

3.2 Some inherent limitations

In many cases, when the notion of index of refraction does not apply, the alternative notion of the effective complex index of refraction is often employed. Formally, it is easy to understand that this index can always be extracted from reflection or transmission spectra, if the medium is non-scattering and has a sufficiently smooth surface. There is no space here to discuss this topic and the reader is invited to consult treatises on the optics of composite materials.

3.3 The mean diffuse external reflectance of a smooth homogeneous material surface

Thus, any calculation and model must be sufficiently general to include all possible cases of optical behaviour. Following the historical approach of Walsh [26] in 1926, we apply an analytical or computational representation of reflectance (or transmittance) expressed by the ‘Mean Reflectance Surface’. That surface is obtained by numerical integration of the Fresnel reflectance formulæ when *n* ranges in [1 ; 6] and *k* in [0 ; 10] and including an attenuation due to roughness

(Gaussian distribution of 'microfacets') influence[2] . The integration covers the complete hemisphere delimited by the local mean tangent plane to a real material surface. For this purpose the illumination is considered as uniformly diffuse over the illuminated real surface and thus does not participate by the mean of a real indicatrix surface to the angular computation. The results obtained completely include those published by Judd in 1942 (for a real index of refraction) and confirmed by Mandelis *et al.* [1990] when they studied the reflectivity of large powders. This generalisation by the use of the computer capabilities shows the possibility of including colour, transparency and gloss for a given state of surface. The radiance of a specular surface lit by a diffuse (isotropic) natural or unpolarised light source is described in more detail in the next section.

4. Polychromy and gilding

To visualise the effects of polychromy and gilding in an architectural situation, we have studied the Galerie des Rois at the cathedral of Notre-Dame de Paris. The Kubelka-Munk model and its variants, commonly used in the industrial formulation of paints and plastics, can also be applied to the materials of artworks. It enables the analysis of the rendering of paints depending on the thickness of the deposited layers. The combination of colouring materials is described here to show how in the medieval ages the visual effect was perfectly managed. That yellowing of the natural light at the beginning of the sunset, quickly turning orange-red at the end, created a kind of aureola behind the heads of the kings' sculptures.

Figure 13: During a cloudy day, a general view of the median part of the Galerie des Rois at Notre- Dame de Paris cathedral. The sculptures were handcrafted during the XIXth century under the restoration campaign led by the famous architect Eugène Viollet le Duc.

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[2]A homogeneous and isotropic roughness [3] distribution only results in an amplitude shift of the reflectance surface while the spectral modification remains very weak.

We believe that special visual effects could have been produced by a caustic phenomenon focusing the solar light on the naked stone and, thus, back-lighting the heads through a multiple scattering process. The complete scientific approach for sustaining our hypothesis is successively described in the following sections. First, we expose the documents and observations known about Notre- Dame de Paris and its polychromy. Second, the 3D acquisition process and treatments of collected data are presented. Third, the measurements made on materials and light are summarised as an introduction to the spectral simulation. Physically-based simulations presented here demonstrate a brand new hypothesis concerning the optical role of gilding in the medieval era. During a cloudy day or under a very directional solar light in the afternoon, the arcatures just above each statue of king are always in shadow. Under very intense direct illumination, the cast shadows have a very high contrast. For softening these hard cast shadows the ancient Greeks painted the wall behind a pediment with a dark blue colour.

4.1 Iconography and polychromy

The facade of the cathedral depicts an iconographic scheme based on the cult and glorification of the Virgin Mary. Originally, the first sanctuary was dedicated to St. Stephen, but the rise of the cult of the Virgin at the beginning of the 13th century and the link made between Mary and the Church explain why the cathedral became 'Notre-Dame de Paris'. Two of the three portals depict stories of the Virgin's mother, St. Anne, and the other scenes from the life of Mary with the Coronation and the Dormition. The third portal deals with the Last Judgement, a theme which is classical in medieval iconography, and particularly on cathedral's portals. The presence of the Galerie des Rois on Notre-Dame de Paris must be related to the cult of the Virgin. The 28 statues under the arcatures actually represent the kings of Judah, from Jess´ to Joseph, according to St. Matthew's Gospel (1:6-16). But a contemporary text named *Les XXIII manières de vilains* spread the seditious idea that the sculptures were representations of the kings of France. Thus when the French Revolution broke out, the statues were all cast down onto the forecourt and soon beheaded.

Nowadays, we see on the facade the 19th century copies made for the restoration supervised by Viollet-le-Duc (Fig. 13), but in 1977 many fragments of 21 of the original heads were found and are now preserved in the Musée National du Moyen Age-Thermes de Cluny in Paris (Fig. 14). Traces of polychromy were found on the remaining heads and also on some details of architecture, as the restoration in 1996-2000 proved [16]. Almost all of the kings wore a full coat open over a dress tight around the waist, crowned and holding a sceptre in the hand. Hair and beard were painted in yellow, blue-grey or red colours, faces were painted in flesh colour with pink highlights on the cheeks, nose and ears. Eyes were surrounded with a black outline and eyelids sometimes shadowed by a red line [14]. Polychrome Kings of Judah were certainly partly gilded, like the statues and representations of the portals.

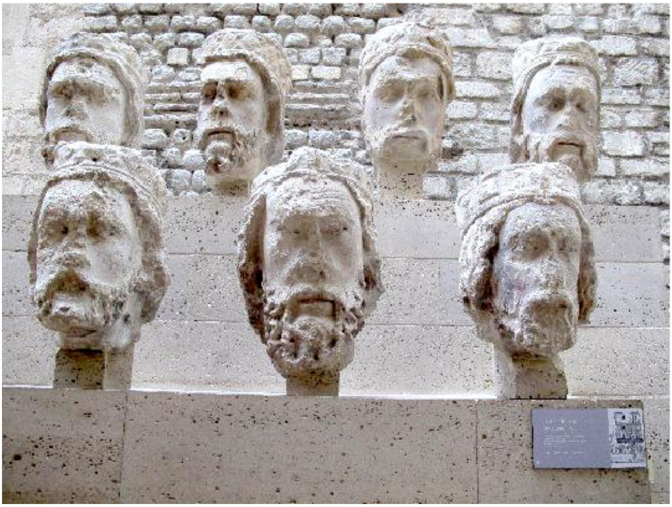

Figure 14: Original remains of the heads of the kings of Judah at Musée National du Moyen-Age in Paris[14]. The medieval heads were not known by E. Viollet-le-Duc and J-B Lassus when they started their restoration. The kings statues were destroyed during the French Revolution and have been retrieved only in 1977.

In addition some polychromy was found in unusual locations, for example in the back of the trefoil arcatures. This can be explained by the projection of sunlight, particularly at sunset. Sunbeams projected onto the facade play with polychromy, emphasising the iconographical scheme. When the portals at ground level were already in darkness, the elevated Galerie des Rois remained colourful, enlightened and gilt, symbolising that the kings, as ancestors of Christ, were protecting Christians. Polychromy in the medieval era was very extensive on monuments. It is well known that all sculpted parts and all ornaments were painted and sometimes gilded. Here the optical role of gold is not only metaphorical because the noble metal is always considered as 'light', 'material' and moreover 'symbol'. Light is always associated with God so that shadows cast by the material world express a complete opposition to the immaterial heavenly realm. Exposed to the assaults of weather, however, the western facade struggled to preserve its polychrome attributes through the centuries. For that meteorological reason, the interior remains of painted parts are more frequently found in their original state without any later restoration.

5. Acquisition of 3D data and colour characterisation

3D data of the facade and statues of the kings was obtained thanks to the knowledge and technique of the Trimble-Mensi company. Laser range scanners were used to capture point clouds, which were resolved using 'Cloud Based Registration' software (RealWorks). The complete 3D point cloud of 52 972 755 elements was partitioned into three subsets: facade, pertinent details (Kings gallery, the three great doors, the Virgin Railings), and the public road. The pertinent details exist in two levels of spatial resolution: 5 and 10 mm. The rest of the monument was acquired at 50 and even 250 mm resolution.

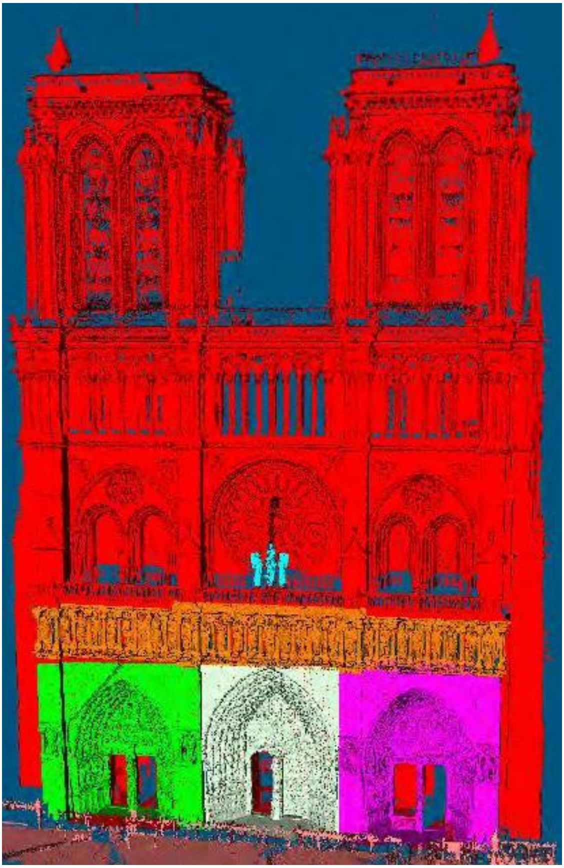

Figure 15: Results of the 3D scanning process obtained by the 2 GX and FX set of sensors. About 53 million of 3D points were acquired.

The project was a collaboration between Trimble-Mensi with the Laboratoire Mathématiques Appliquées aux Systèmes (ECP) and a medievalist teacher working at the École du Louvre. This interdisciplinary study enabled exploration of an historical hypothesis about the possible optical role of gilding. In the Middle Ages, the symbolism of colours, their associations and even the order of interpretation (from back to front in the painted image) were very different from what they are today [23]. Lighting conditions, inside or outside, were completely different too. The spectrum of the prevailing natural light is not known and the colour classification does not match at all our occidental orthodoxy of today. As colours were regarded more as symbols than materials they could not be disposed at random nor mixed together. Thus it is very surprising to find a mixture of pigments on a sculpture from that period [13,15]. The discovery of some traces of a reddish substance on the inner surface (intrados), as shown in Figure 16, was the starting point of questioning the special optical role of gold [16].

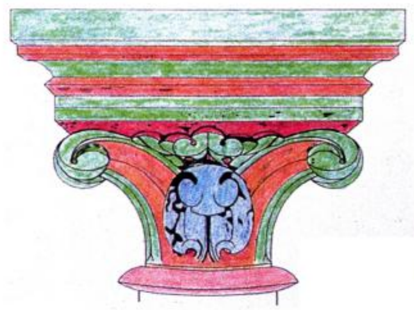


Figure 16: Left: The retrieved polychromy composition and disposal of a typical capital situated on the western facade. Figure extracted from [16]. Right: The remains of a mixture of reddish pigments made of ochre and red lead over a white lead (ceruse) preparation layer. Images extracted from [16].

5.1 Light effects

Colour was much more defined by a certain sensibility to light in the Middle Ages, and sometimes translators used modern words to translate what in the Latin texts was more an impression of light than a 'real' colour [23]. This idea of light was also emphasised by contemporaneous writers, like Hugues de Saint-Victor (1096-1141) in his *De tribus diebus*: "What is more beautiful than light which is without colour but colours everything by lighting?"[11]. And there was also an allusion to "God Who is Light". Thus the remains of polychromy have to be reinterpreted thanks to iconography and effects of sunlight on the fronts of cathedrals. Viollet-le-Duc noticed that the facade of Notre-Dame de Paris was covered by numerous painted and gilded parts such as the Galerie des Rois, all coloured and gilded. A few sentences later, he explains a medieval technique consisting in placing glass sheets (bullion technique) covered with gold or tin sheets behind the painted sculpture [2]. In a future work we shall consider the effect of such mirrors on the solar reflected light near sunset and especially for the longest day. The summer solstice gives today a frontal illumination almost perpendicular to the western facade. Due to the precession of the equinoxes the angular relationships between a monument and the solar disk position in the sky for a given date may have been slightly different from what it is today. That angular annual shift is about 50" per year. Such an effect implies a rotation of 12° over 850 years. One can imagine an extraordinary visual effect at sunset in summer as the sun is moving at a high speed in the sky, progressing by its own angular diameter each couple of minutes so that the reflected beam produced by each small mirror facet changed in 'real time' (Fig. 17).

5.2 Bidirectional reflectance distribution function and colour

As we always need to characterise what is, for a given angle of observation, the visual appearance of a material for any illumination conditions, we designed a special way of describing the Bidirectional Reflectance Distribution Function (BRDF). This spectral function, expressed in sr^{-1}, connects the reflected radiance for a given direction of observation to the incident irradiance from a particular direction. As such a general function, for opaque materials, when sufficiently sampled, requires a large volume for storage, the computer graphics community has developed many physico-mathematical models. In our approach, the generating functions always used the complex index of refraction, described by the two spectral functions (n,k), and we then express all the physical laws with these functions. BRDF enables us to compute accurately the light transport between surfaces to render a 3D scene in photorealistic style. Given a BRDF for a particular material, we have to convert its output into colour coordinates via the tristimulus transformation using a colorimetric standard observer. BRDF is very delicate to measure and well adapted for homogeneous materials but generally needs to be approximated for textured surfaces. We can also compute predicted BRDF for complex materials such as goniochromatic surfaces, but the human visual system is always the judge of closeness of the rendered model to the real surface, photographed in a light booth with a calibrated digital camera. The simulated optical properties of a material object are then displayed alongside the photo of the object on the same screen.

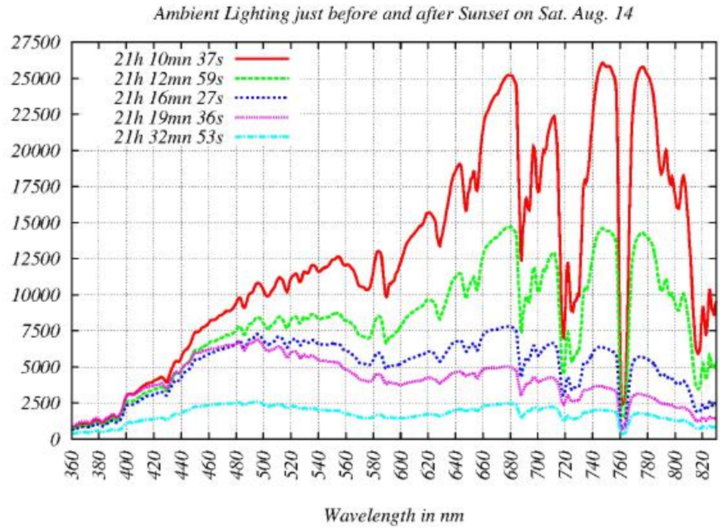

Figure 17: Rapid evolution of the solar spectrum during sunset on 2010 Aug. 14th at Pointe de Trévignon (Brittany).

6. Optical simulation and modelling

Art materials and more precisely painting materials and their properties have been studied for a long time [1,5]. In computer graphics, the simulation of pigmented media such as paints and plastics is not new [17, 6] but is always making progress thanks to spectrophotometry using portable devices [9,7]. Painting materials during the Middle Ages are not very different from those known in Antiquity. The classical organisation of the paint layers deposited for polychromy are, from the innermost to the outermost: stone, white lead (ceruse = lead carbonate) or gesso (gypsum and/or chalk mixture), first pigmented layer, final pigmented layer. The first paint layer is not always present but, generally it fulfils an economic function.

For painting a king statue the most expensive and precious pigments were used by the artist and disposed in the thinnest external layer. However to reinforce the optical effect of those noble pigments the artist did not cover the white lead preparation layer with the most expensive material first. For example, a painting with natural ultramarine blue (the most precious pigment) will be optically more efficient if deposited over a first layer that is already blue (e.g. an azurite layer). Some recent works had given us experience in modelling the influence of the bole colour on the visual appearance of gilded statues [12]. The model use the inner coating including successive layers of ceruse while the bole layer made of a mixture of 50% red lead and 50% red ochre on which was deposited the gold leaf fixed by an organic adhesive substance. That last metallic layer was described from our knowledge of the medieval techniques of gilding. Note that to obtain a good shining effect with a golden surface, convex shapes need to be disposed in such a way that the solar light strikes them and plays with multiple inter-reflections. Here, on the contrary, the golden parts are concave and orientated for receiving the light only at sunset. Thus a moderate porosity and distribution of cracks inside the gold leaf itself was incorporated in the simulations. The influence of the coloured preparation (bole) on the visual appearance of metallic gold leaf was modelled according to the Kubelka-Munk theory of pigments mixing in paint layers. The multiple internal scattering of light inside the bole layer gives a slightly reddish tone to the gilt. The metallic influence was rendered by a model including measured roughness and incorporating at each step of the computation the polarisation state of light (Fig. 18).

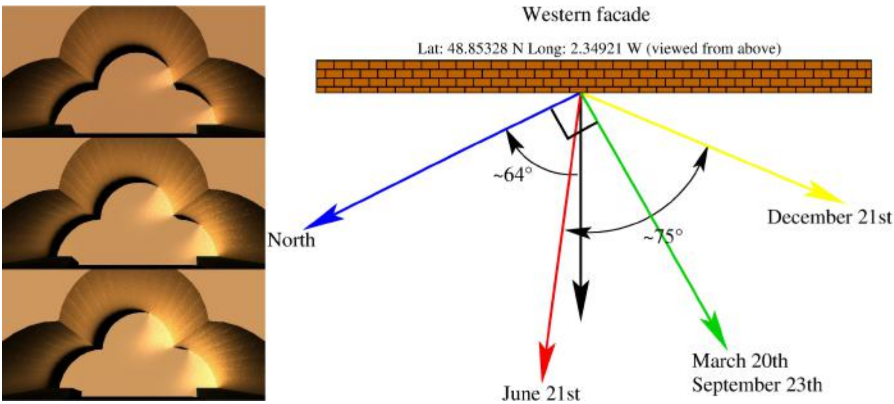

Figure 18: Left: Schematic evolution of the back lighting of the kings heads due to caustics formation by reflection of the solar light at sunset. The solar light is guided by a grazing and progressive reflection on the gilded cylindrical shapes of the arcatures. From top to bottom the solar spectrum slightly changes according to the spectrophotometric emission measurements made from 21:09 to 21:33 on 2009 July 15th. Right: The angles considered for the natural direct lighting by the sun today. Due to the precession of the equinoxes (50" per year, i.e. 12°) we can suppose that for 850 years a correction of the illumination gave frontal lighting for the 21st June. The cathedral was built on the remains of a roman temple and could have maintained the initial orientation taking 'political' advantage of the symbolism of light.

6.1 Natural lighting

Sunset was at 21:49 on 2009 July 15th and we measured many emission spectra with the aid of a MAYA 2000 spectrophotometer. The experimental setup gave very detailed information on the yellowing and amplitude of the solar lighting. For normalisation a first measurement was made at a fixed angle between the collecting optical fibre and the reference white orientated in the direction of the centre of the solar disc. All measurements were obtained in similar conditions with the same geometrical observation parameters (Fig. 17). The reflectivity and transmissivity functions governed by the Fresnel formulas depend mainly on the imaginary part of the index of refraction defined in eq. (1). That effective index of refraction [10] has a lower imaginary part than pure gold and consequently its spectral transmissivity is magnified. In this way the fraction of light originating in multiple scattering inside the volume of the bole layer and diffusely emerging through the metallic leaf is accessible.

The effective medium formed by the metallic film and the nanoscopic holes was approximated by a Maxwell-Garnett model of the dielectric function. Using the diffuse reflectance factor spectrum of the bole, considered as a completely opaque and multiply scattering pigmented medium, we could add this contribution to the global reflectance factor. The main fraction of the reflected light is governed by Fresnel laws modulated in amplitude by an optical roughness. The roughness influence is extracted from laser profilometric measurements. The diffuse reflectance factor emerging through the metallic leaf and from the bole was obtained by the Kubelka-Munk model for opaque layers whose parameters (the K / S factors) are inverted from the diffuse reflectance factor R_∞ of the bole. We thus consider an effective dielectric-metallic medium made of a metallic continuum embedding a resin composed of egg white holes and air. The model leads to the computation of an effective index of refraction for the surface plasmon component depending on the relative concentration of holes and cracks.

6.2 Rendering with Virtuelium

Spectral rendering has a relatively long history in the computer graphics community [18]. The radiative transfer equation has been algorithmically solved for diffuse materials with the radiosity rendering software [21]. Another category of global illumination algorithms was proposed by Jensen at the end of the 90s. For combining the advantages of a spectral ray-tracing rendering and radiosity-like light transport effects, we used the *Virtuelium* open-source software to simulate the natural lighting on the facade of Notre-Dame de Paris at sunset (Fig. 19).

Some of the main characteristics of *Virtuelium* are:

1. Spectral rendering with adjustable wavelength sampling for all elements (viewer, materials BRDF, illuminants, propagation media);
2. Polarisation effects accounted for at all steps of the computation;

3. XML formalism for the 3D scenes and data descriptions;
4. Multi-threading and/or grid computing;
5. Multi-layered materials (macroscopic layers of paints or assembly of multiple thin films);
6. High dynamic range (HDR) imaging;
7. Spectral Texture Mapping. A stochastic photon-mapping algorithm according to Jensen [19, 20] is implemented for realistic rendering of the global illumination in a 3D space.

Virtuelium accounts for the most intrinsic characteristics of materials, including complex indices of refraction, and parameters describing how the materials are used in the 3D simulation (powder, bulk, mixture, solution, composite, etc). The computed images are thus based on the optical properties measured on all the material remains found on the sculptures, referred to the CIE 1964 (10°) colorimetric observer. Notice that, according to the real process of painting on stone, the coloured effects obtained by simulation are quite different from a polychrome restitution by a direct electric lighting on the real historic building (and, paradoxically, during the night).

7. Conclusions

From transparency to opacity, from refraction and dispersion of light to metallic reflection, from single scattering to multiple scattering either in surface or volume physically based rendering offers a good
approach to visualise objects in their original state or in a future state of their existence. There are still many things to do for achieving and, why not, for viewing a complete proposition of the polychromic splendour of Notre-Dame de Paris cathedral. Spectral simulation with *Virtuelium* software offers capabilities that enable a visualisation of the appearance of the polychrome of many historic buildings very different from how we perceive it today. We have shown that a scientific visualisation founded on spectral simulation of the interaction of light with materials greatly helps in the understanding of what could be the medieval design of the symbolism of light. Their method of using gold to guide the natural light to soften the cast shadows behind the statues could be useful to enhance the electric lighting of today for our monuments during dark hours. New visualisation devices of large dimensions can create immersive sensations that modify our global perception and, accordingly, the applicable scientific models.

Figure 19: The Kings Gallery of Notre-Dame de Paris. Simulation obtained with Virtuelium using the photon-mapping algorithm, the optical model for gilts and the polychromy indications that were available. The lighting is estimated on the basis of a solar emission spectrum progressively modified by a Rayleigh scattering model. The sun is placed, thanks to a script (python language), and is one of the the pertinent parameters given to Virtuelium together with power, spectrum, direction, etc. The computation, involving 120 Intel processors, for a high resolution rendering is made each 3 minutes from 20:58 to 21:58 for the ideal illumination on 2012, June 21st (Images for 20:58 and for 21:55). The computation time was about 40 minutes! Notice the place where the caustics are shining just behind and above the heads. Notice that the red and blue clothes are arbitrarily distributed here but in a plausible way. The colours of any other ornaments are rendered according to the historical indications[16] given in Figure 16.

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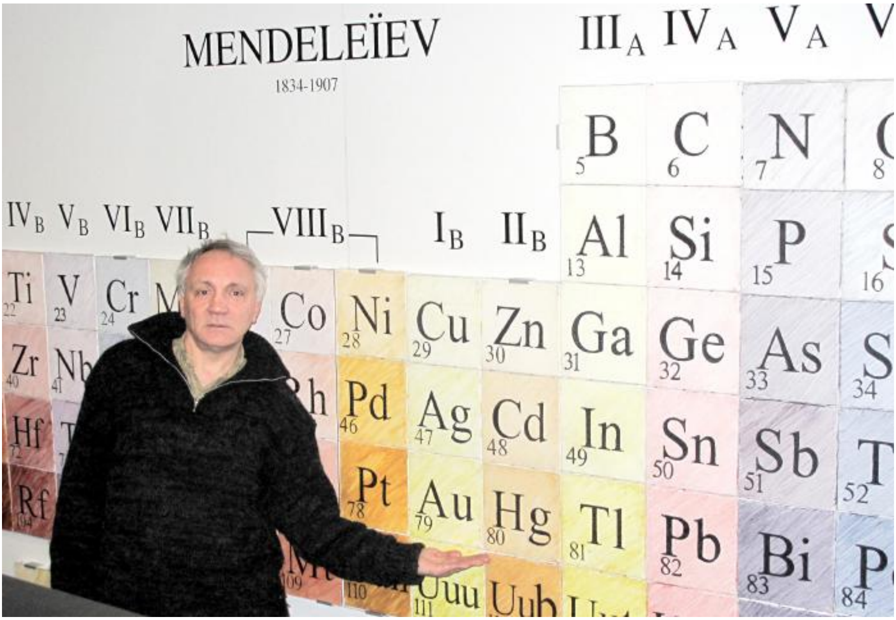
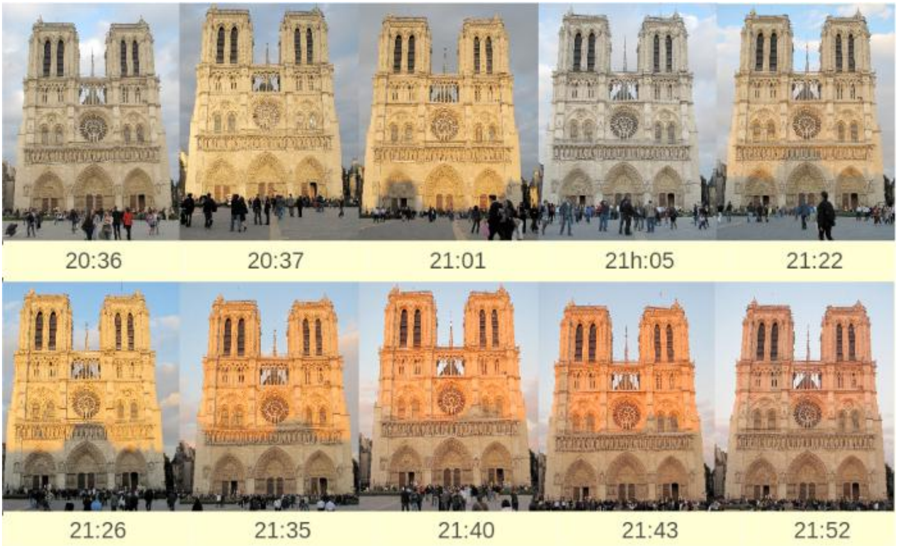
❦

Dr Patrick Callet indicates metals in Groups10 and 11 of the periodic table with which he has been particularly concerned in his researches.





The 'Sun Day,' natural lighting at sunset on Notre-Dame de Paris on 21st June 2010. (photos: P. Callet)

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